# Applecross Senior High School

### Semester One Examination, 2020

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**SPECIALIST**

**UNIT 1**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

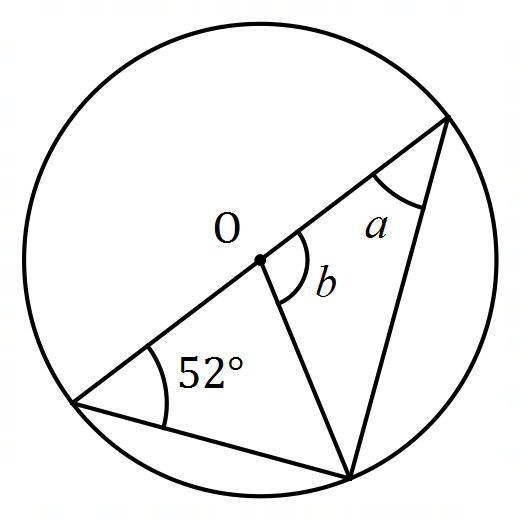
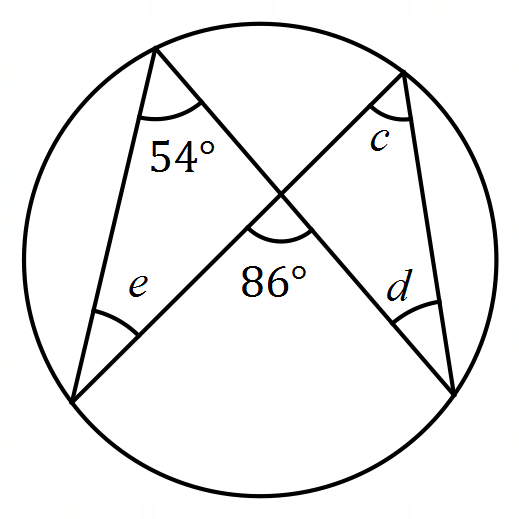
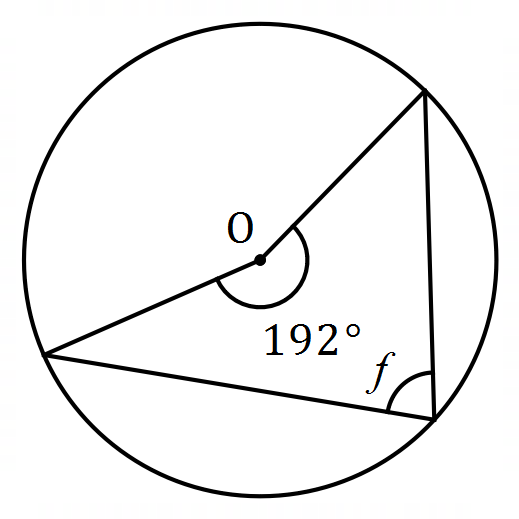
Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (6 marks)

Determine the size of the angles marked and shown in the circles below. Where marked, is the centre of the circle.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ each correct angle |

Question 10 (5 marks)

Three forces act on an object so that it remains in equilibrium. Two of the forces have magnitudes of N and N and the angle between their directions is . Determine the magnitude of the third force and the angle its direction makes with the smaller force.

|  |
| --- |
| **Solution** |
| Hence angle between directions is . |
| **Specific behaviours** |
| ✓ diagram showing vector sum is zero   uses cosine rule to solve triangle   magnitude of resultant   uses sine rule   direction with smaller force |

Question 11 (8 marks)

(a) An art gallery plans to display a single painting on each of the three walls in a room. Determine how many arrangements of paintings are possible in the room if they have a selection of different paintings to choose from. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates method   correct number of arrangements |

(b) In another room, the gallery plan to hang different paintings in a row. If of the paintings are by the artist Marr, determine the number of different arrangements of paintings that are possible when

(i) the paintings by Marr must be at the ends. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses   correct number of arrangements |

(ii) the paintings by Marr must be next to each other. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ groups Marr together   correct number of arrangements |

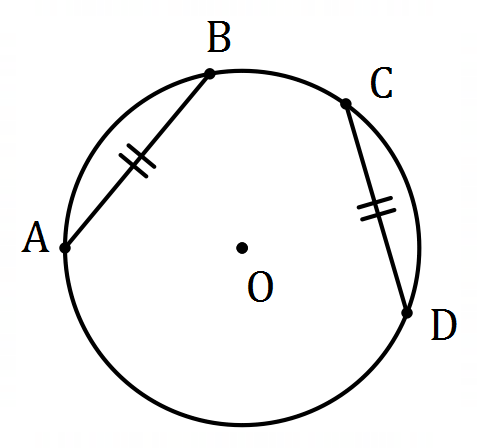
(iii) the paintings by Marr must be apart and not at the ends. (2 marks)

|  |
| --- |
| **Solution** |
| non-Marr leave spaces to hang Marr in between (N\_N\_N\_N\_N\_N): |
| **Specific behaviours** |
| ✓ indicates method   correct number of arrangements |

Question 12 (8 marks)

(a) Prove that chords of equal length subtend equal angles at the centre of a circle.

(3 marks)



|  |
| --- |
| **Solution** |
| (given)  (all radii)  Hence (SSS)  Hence - chords of equal length subtend equal angles at the centre. |
| **Specific behaviours** |
| ✓ establishes congruency of sides   establishes congruency of triangles   concludes equal angles |

(b) Points and lie on a circle of radius cm so that cm. Determine

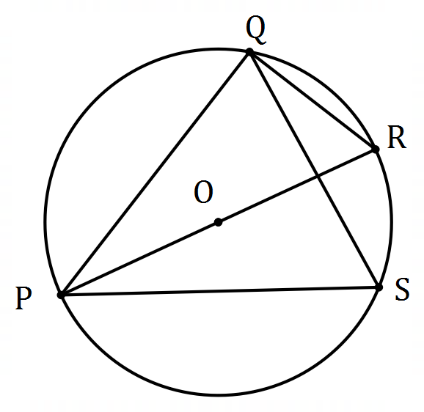
(i) the distance of chord from the centre of the circle. (3 marks)

|  |
| --- |
| **Solution** |
| Let midpoint of chord be . Then |
| **Specific behaviours** |
|  uses/defines midpoint or sketch diagram  ✓ indicates correct method   correct distance |

(ii) the angle subtended by chord at the centre of the circle. (2 marks)

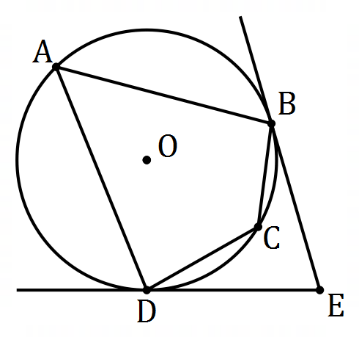
|  |
| --- |
| **Solution** |
| Let (half angle required). Then |
| **Specific behaviours** |
| ✓ indicates correct method   correct angle |

Question 13 (7 marks)

(a) The diagram shows points and that  
lie on the circumference of a circle centre .  
 is a diameter and the size of .

Determine, with reasons, the size of . (3 marks)

|  |
| --- |
| **Solution** |
| (angle in semicircle)  (angle sum in triangle)  (angles on same arc) |
| **Specific behaviours** |
| ✓ uses angle in semicircle   uses angle sum in triangle   correct size of angle, with reason |



(b) In the diagram shown, and are points  
on the circumference of a circle with centre .

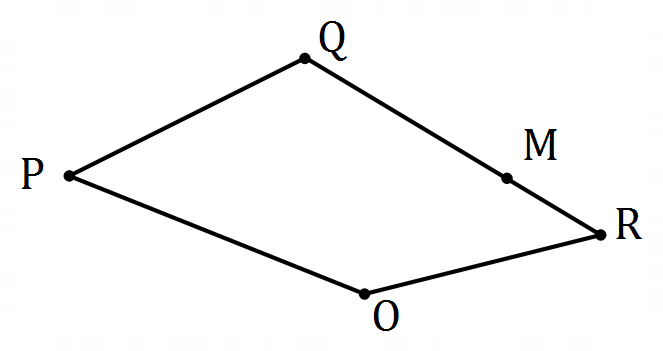
Tangents to the circle at and intersect at .

Determine, with justification, the size of   
when . (4 marks)

|  |
| --- |
| **Solution** |
| (radius-tangent angle)  (angle sum of quadrilateral )  (centre-circumference angles)  (opposite angles in cyclic quadrilateral) |
| **Specific behaviours** |
| ✓ uses radius-tangent angle   correct   uses angle at centre-circumference   correct angle |

Question 14 (8 marks)

In quadrilateral shown below, lies on so that .



(a) If and , express the following in terms of and/or .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct expression |

(i) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses correct vector notation   correct expression |

(ii) . (2 marks)

(iii) . (2 marks)

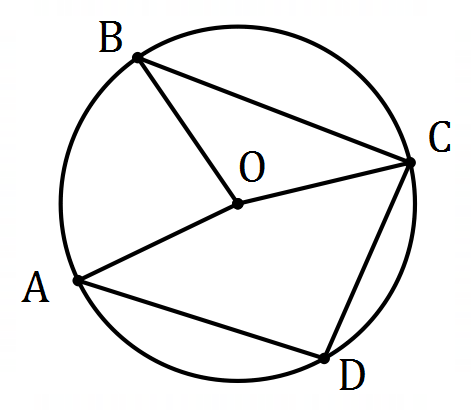
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates suitable vector sum   correct expression |

(b) If is the origin and points and have coordinates and respectively, determine the distance . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ substitutes into expression for     correct magnitude |

Question 15 (8 marks)

(a) The vertices of quadrilateral lie on the circumference of a circle centre shown below. Given that and , determine with reasoning the size of angle . (4 marks)



|  |
| --- |
| **Solution** |
| (angles at centre-circumference)  (adjacent angles)  (isosceles triangle) |
| **Specific behaviours** |
| ✓ uses angles at centre-circumference   uses adjacent angles   uses isosceles triangles   correct angle |

(b) The vertices of triangle lie on the circumference of a circle. Given that cm, cm and cm, prove by contradiction that is not a diameter of the circle.

(4 marks)

|  |
| --- |
| **Solution** |
| Assume that is a diameter of the circle, so that the angle in a semicircle theorem implies that must be right angled at .  If is right angled, then Pythagoras' theorem implies that .  But and .  This result contradicts our assumption that is a diameter and so cannot be a diameter of the circle. |
| **Specific behaviours** |
| ✓ states assumption and uses angle in semicircle theorem   uses Pythagoras' theorem to state relationship between side lengths   shows relationship is false   explains contradiction |

Question 16 (7 marks)

(a) A calculator can generate random integers between and . Use the pigeonhole principle to explain why random integers should be generated to be certain that at least of them are the same. (3 marks)

|  |
| --- |
| **Solution** |
| There are pigeonholes (integers from to ) and each random integer produced is a pigeon.  By the pigeonhole principle:  If only integers are produced, there will be at least pigeons in at least one pigeonhole, but if integers are produced then there will be at least pigeons in at least one pigeonhole.  Hence integers should be produced to be certain that at least of them are the same. |
| **Specific behaviours** |
| ✓ defines pigeonholes   shows insufficient   shows sufficient |

(b) customers bought a total of items from a supermarket. Given that each customer bought at least one item, show that at least two of the customers bought the same number of items. (4 marks)

|  |
| --- |
| **Solution** |
| Assume that each customer bought a different number of items.  Then the minimum number of items bought would be:  But the number of items bought () was less than this minimum, which contradicts the assumption made.  Hence at least two customers bought the same number of items. |
| **Specific behaviours** |
| ✓ states assumption   uses assumption to calculate minimum   states contradiction   summary statement |

Question 17 (9 marks)

(a) Determine the scalar product of

(i) and . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(ii) two vectors with directions apart that have magnitudes of and . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(b) Given that and simplify . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expands using scalar products   simplifies using magnitudes   correct value |

(c) The position vectors of points and are and . Show use of a vector method to determine the size of angle . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ vectors and   shows magnitudes   shows scalar product   correct angle |

Question 18 (8 marks)

A school yearbook is produced by a committee of teachers and students. teachers and students have nominated for the committee.

(a) Determine how many different committees could be formed from the nominations.

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ chooses teachers and students separately   correct number |

(b) The student nominations include two sets of twins. Determine how many different committees could be chosen that include at least one set of twins. (4 marks)

|  |
| --- |
| **Solution** |
| Choose students with at least one set of twins (Set A, Set B, Others):  Ways to choose whole committee: |
| **Specific behaviours** |
| ✓ indicates isolation of cases   uses systematic approach   correct ways to choose students   correct number of committees |

(c) Suppose one of the teachers in the committee will be appointed as treasurer and one of the students will be appointed as secretary. Determine how many different committees can be formed with this structure. (2 marks)

|  |
| --- |
| **Solution** |
| Select a teacher and others, select a student and others: |
| **Specific behaviours** |
| ✓ indicates correct method   correct number |

Question 19 (8 marks)

Oil platform T lies km away from another oil platform F on a bearing of . A steady current of km per hour flows between the platforms on a bearing of . A small boat at F, with a cruising speed of km per hour, needs to arrive at T by pm.

Determine the bearing that the boat should steer and the latest time it should depart from F.

|  |
| --- |
| **Solution** |
| If journey takes hours, then and .  using sine rule:  Hence and .  Bearing to steer:  Distance using sine rule:  Hence steer on bearing and leave before . |
| **Specific behaviours** |
| ✓ sketch diagram   angle at T   equation using sine rule   solves angles in triangle   solves for second side in triangle   journey time in hours   correct time to leave   correct bearing |

Question 20 (8 marks)

Circles and intersect at points and . passes through , the centre of . lies on so that line segment is tangential to at . Let .

(a) Sketch a diagram to show the above information. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ passing through centre of   tangent   marks angle |

(b) Determine in terms of . (1 mark)

|  |
| --- |
| **Solution** |
| (angle centre-circumference ) |
| **Specific behaviours** |
| ✓ correct expression |

(c) Explain why . (1 mark)

|  |
| --- |
| **Solution** |
| (alternate angles ) |
| **Specific behaviours** |
| ✓ states equal to using circle theorem |

(d) Prove that . (3 marks)

|  |
| --- |
| **Solution** |
| (exterior - interior sum in )  is isosceles  Hence . |
| **Specific behaviours** |
| ✓ deduces with reason   states isosceles   deduces lengths equal |

Question 21 (8 marks)

Particle , initially at the point with position vector cm, moves with a constant velocity of cm/s. Particle is stationary at the point with position vector .

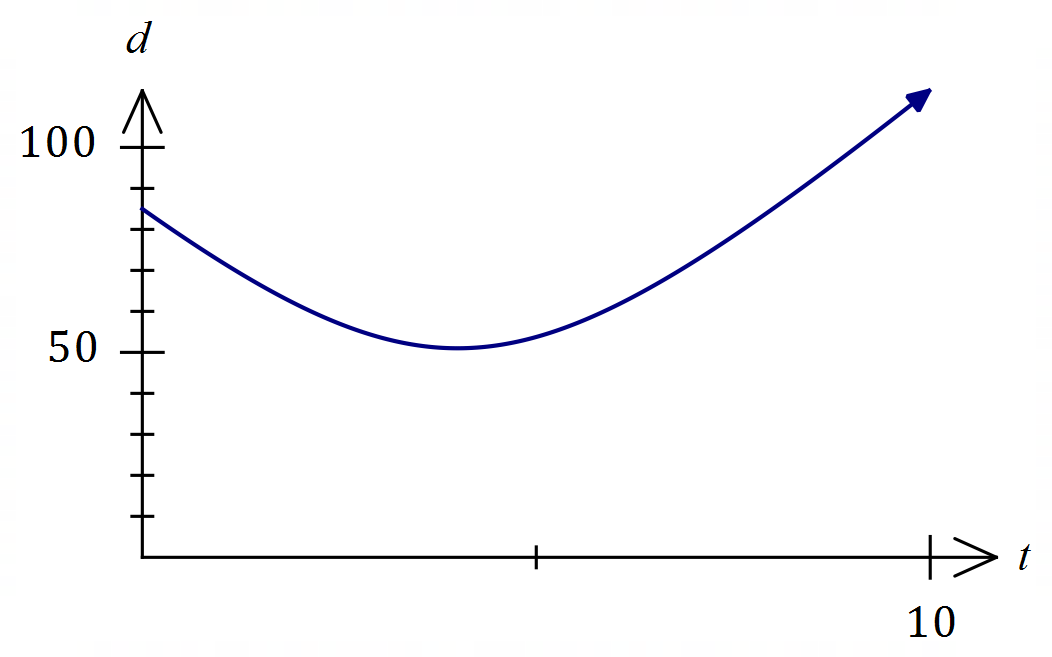
(a) Determine the initial distance of from . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ vector   correct distance |

(b) Determine an expression for the distance between and after seconds. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ position vector for at time  ✓ vector   distance expression (no need to simplify) |

(c) Sketch a graph of against and hence determine the time that minimises and state what this minimum distance is. (3 marks)



|  |
| --- |
| **Solution** |
| Minimum when s  Minimum distance is cm |
| **Specific behaviours** |
| ✓ sketch graph   time   minimum distance |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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